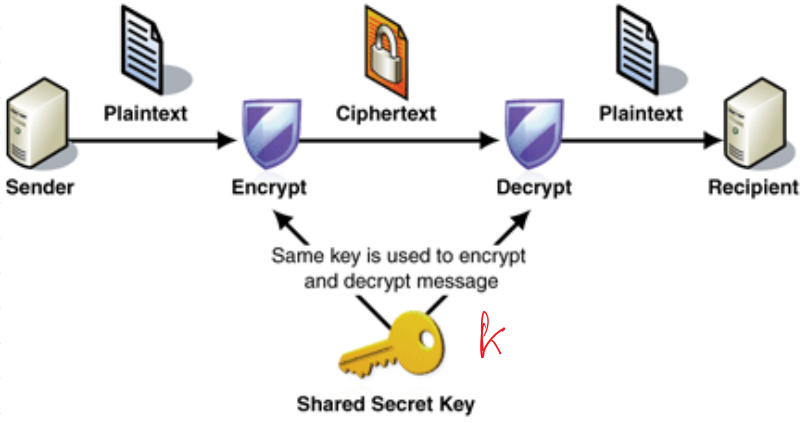
Secure channel **https://** PROTOCOL
https://www.swedbank.lt/private



Let p is prime, e.g. $p=11$.

$\mathbb{Z}_p^* = \{1, 2, 3, \ldots, p-1\}$ $*\bmod p$; $:\bmod p$

$\mathbb{Z}_{11}^* = \{1, 2, 3, \ldots, 10\}$ $*\bmod 11$

$2 \cdot 6 = 12 \bmod 11 = 1$

$12 - 11 = 1$; $11 / 1$

$a:b = a * b^{-1} \bmod p$

$b * b^{-1} = 1 \bmod p$

Multiplication Tab Z_{11}^*										
*****	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	2	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	2	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

$2^{-1} = 6$

$3^{-1} = 4$

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

$a / b \bmod 11 = a \cdot b^{-1} \bmod 11 \Rightarrow$ find $b^{-1} \bmod 11 \Rightarrow b^{-1} \cdot b = 1 \bmod 11$

$3/5 = 0.6$; $3/5 = 3 \cdot 5^{-1} \bmod 11 = 3 \cdot 9 \bmod 11 = 5$

Base values

Exponent function in $\mathbb{Z}_p^*$: $a = g^x \bmod p$

Exponent
Tab Z_{11}*

^	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1
2	1	2	4	8	5	10	9	7	3	6	1
3	1	3	9	5	4	1	3	9	5	4	1
4	1	4	5	9	3	1	4	5	9	3	1
5	1	5	3	4	9	1	5	3	4	9	1
6	1	6	3	7	9	10	5	8	4	2	1
7	1	7	5	2	3	10	4	6	9	8	1
8	1	8	9	6	4	10	3	2	5	7	1
9	1	9	4	3	5	1	9	4	3	5	1
10	1	10	1	10	1	10	1	10	1	10	1

Exp values

$2^4 = 16 \bmod 11 = 5$

16 | 11
−11 | 1
5

Fermat theorem: if p is prime then

$z^{p-1} = 1 \bmod p$

$p-1 \equiv 0 \bmod (p-1)$

p−1 | p−1
−p−1 | 1
0

$s = (u + x\,h) \bmod (p-1) \rightarrow S = \sigma = (r, s) \leftarrow (v, r, s)$

Discrete Exponent Function - DEF:

x	0	1	2	3	4	5	6	7	8	9	10	$\mathcal{I}_{10}$
$2^x \bmod 11$	1	2	4	8	5	10	9	7	3	6	1	$\mathcal{I}_{11}^*$

0	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	1

Let p is prime $\Rightarrow \mathcal{I}_p^* = \{1, 2, 3, \ldots, p-1\}$

$x \in \mathcal{I}_{p-1} = \{0, 1, 2, 3, \ldots, p-2\}$

$+ \bmod (p-1)$ $- \bmod (p-1)$

$* \bmod (p-1)$

$\Gamma = \{2, 6, 7, 8\}$; $|\Gamma| = 4$ generators of 10 elements of $\mathcal{I}_{11}^*$.

Number of generators is about 40% of $\mathcal{I}_p^*$.

C.5.3 **Finding generators**.

We have to look inside Z_p* and find a generator. How?

Even if we have a candidate, how do we test it?

The condition is that ***g*** is a generator of Z_p* which would take $|Z_p^*|$ steps to check.

In fact, finding a generator given ***p*** is in general a hard problem.

In fact, even checking that ***g*** is a generator given ***p*** is a hard problem.

But what we can exploit is that is **strong prime *p*=2*q*+1** with ***q***=(***p***-1)/2 prime.

Note that the order of the group Z_p* is ***p*-1=2*q***. Prime ***p*** is called a **strong prime**.

11

$23 = 2 \cdot 11 + 1$

$27 = 2 \cdot 13 + 1$

But what we can exploit is that is **strong prime** $p=2q+1$ with $q=(p-1)/2$ prime.
Note that the order of the group Z_p^* is $p-1=2q$. Prime p is called a **strong prime**.

Fact C.23. Say $p=2q+1$ is **srong prime** where $q=(p-1)/2$ is prime. Then g in Z_p^* is a generator of Z_p^* iff (if and only if - tada ir tik tada) $g^q \neq 1 \bmod p$ and $g^2 \neq 1 \bmod p$.

```
>> p=int64(genstrongprime(28))
p = 251487959
>> q=(p-1)/2
q = 125743979
>> isprime(q)
ans = 1
```

>> mod_exp(g, q, p) ≠ 1 >> mod_exp(g, 2, p) ≠ 1

Finding generators: g=2; g=3; ...

Public parameters generatiom 28 bits arithmetics

```
>> p =  268 435 019;  % 2^28 -1 --> >> int64(2^28-1)
                      % ans = 268 435 455
>> g=2;
```

```
>> p=int64(268435019)
p = 268435019
g=2;
```

$\mathcal{I}_p^* = \{1, 2, 3, \ldots, p-1\}$; $\mathcal{I}_{11}^* = \{1, 2, 3, \ldots, 10\}$; $* \bmod p$

In Octave $* \bmod p$ is realized by the function for all $a, b \in \mathcal{I}_p^*$

```
>> mod(a*b, p)
```

$u \leftarrow rand(Z_p^*)$

$g^u \bmod p = t_A$

t_A

t_B

$v \leftarrow rand(Z_p^*)$

$t_B = g^v \bmod p$

$k_{AB} = (t_B)^u \bmod p =$

$= (g^v)^u \bmod p = g^{vu} \bmod p$

$k_{BA} = (t_A)^v \bmod p =$

$= (g^u)^v \bmod p = g^{uv} \bmod p$

$k_{AB} = k = k_{BA}$

Diffie-Hellman Key Agreement Protocol (DH KAP)

For KAP parties are using public parameters **PP** = (p, g).
Parties computes temporary open session parameters t_A and t_B in the same way as users public keys are computed.

1. Alice generates secret random number $u \leftarrow rand(Z_p^*)$ and computes
   $t_A = g^u \bmod p$.
   Alice sends t_A to the Bank using any open channel.
2. Bank generates secret random number $v \leftarrow rand(Z_p^*)$ and computes

$t_B = g^v \bmod p$.

Bank sends t_B to the Alice using any open channel as well.

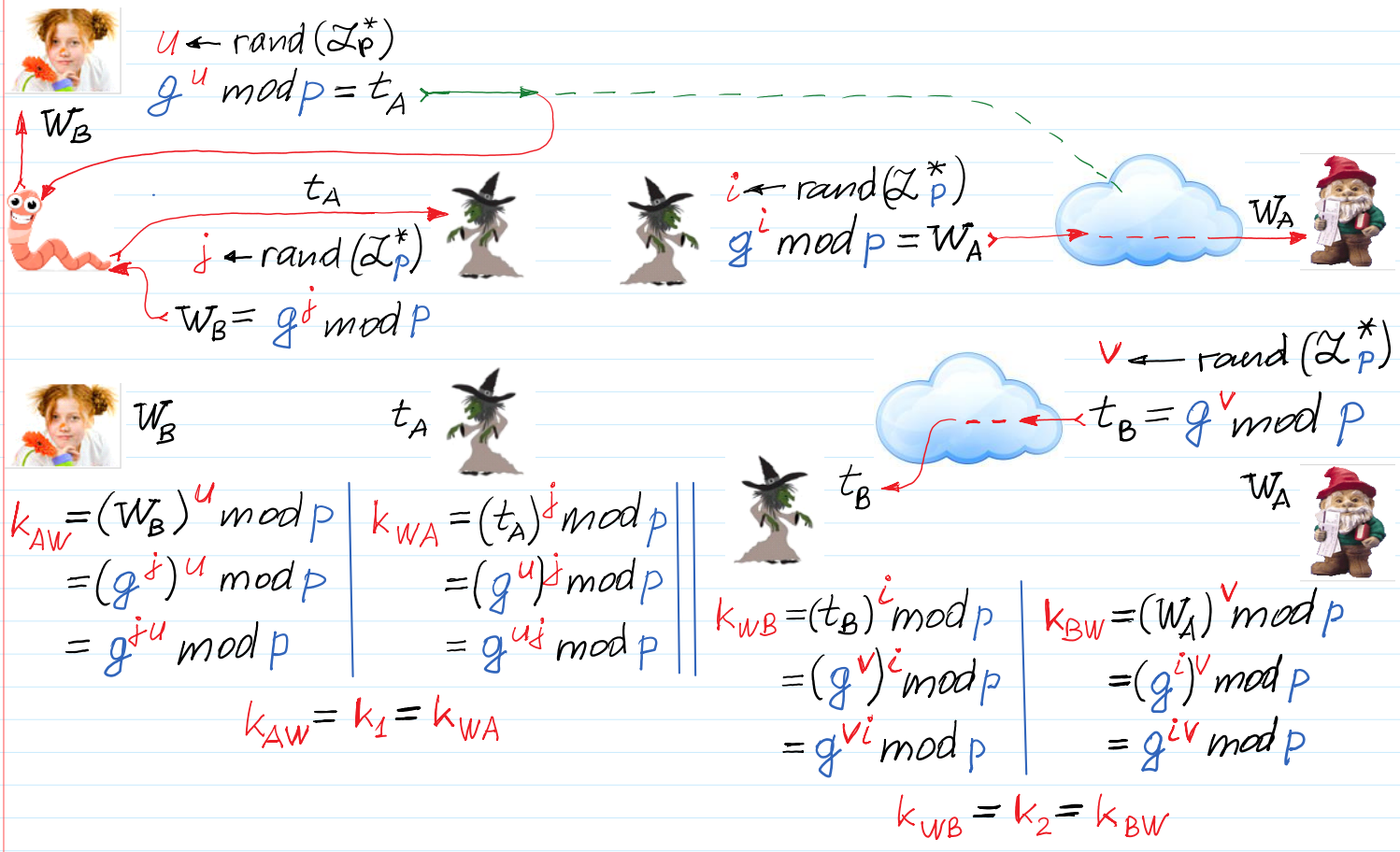
3. Parties independently computes common secret keys. Alice having secret random number u computes k_{AB} and Bank having secret random number v computes k_{BA} in the following way:

$k_{AB} = (t_B)^u \bmod p = (g^v)^u \bmod p = g^{vu} \bmod p$.
$k_{BA} = (t_A)^v \bmod p = (g^u)^v \bmod p = g^{uv} \bmod p$.

Since $g^{vu} \bmod p = g^{uv} \bmod p$ then parties agreed on the common secret key k, where
$k_{AB} = k = k_{BA}$.

KAP is using in **https://** protocol in general and especially in e-banking system.
After the common secret key k agreement parties creates secure channel where information exchanged by Alice and Bob's Bank is encrypted using the same symmetric encryption scheme and agreed common secret key k.

Man in the Middle (MiM) Attack



It is an example of very actual so far kind of active attack directed to KAP. The actuality of this attack is remains high due to the lack of identification from the ordinary customer side. According to this scenario the protocol is executed in the following way.

Alice chooses at random $u \leftarrow rand(Z_p^*)$, computes

$t_A = g^u \bmod p$,

and sends t_A thinking that it is sent to Bob but actually it is sent to Zoe.

Zoe after receiving t_A from Alice chooses at random $j \leftarrow rand(Z_p^*)$, computes

$W_B = g^j \bmod p$,

and sends W_B to Alice thus impersonating Bob.

Alice and Zoe after receiving t_A and W_B computes their secret keys k_{AW} and k_{WA} respectively.

$k_{AW} = (W_B)^u \bmod p = (g^j)^u \bmod p = g^{ju} \bmod p$.

$k_{WA} = (t_A)^j \bmod p = (g^u)^j \bmod p = g^{uj} \bmod p$.

Analogously to and Alice and Zoe agreed on the same secret key

$k_{AW} = k_1 = k_{WA}$.

Zoe continues computations with Bob in the similar way. Zoe chooses at random $i \leftarrow rand(Z_p^*)$, computes

$W_A = g^i \bmod p$,

and sends W_A to Bob thus impersonating Alice.

Bob does not suspecting any badness, as usual, chooses at random $v \leftarrow rand(Z_p^*)$, computes

$t_B = g^v \bmod p$,

and sends t_B to Zoe thinking that he have sent it to Alice.

Zoe and Bob after receiving t_B and W_B computes their secret keys k_{WB} and k_{BW} respetively

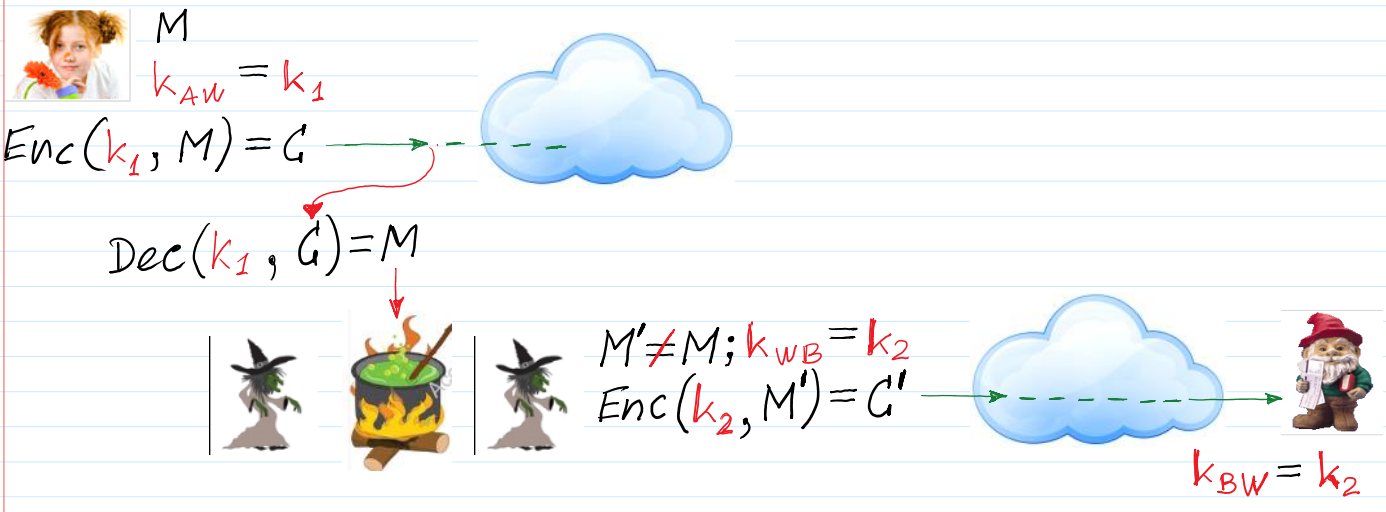
$k_{WB} = (t_B)^i \bmod p = (g^v)^i \bmod p = g^{vi} \bmod p$.

$k_{BW} = (W_B)^v \bmod p = (g^i)^v \bmod p = g^{iv} \bmod p$.

And again, analogously to and Zoe and Bob agreed on the same secret key.

$k_{BW} = k_2 = k_{WB}$.

As an outcome of MiM Attack parties have agreeded two secret keys: key k_1 between Alice and Zoe and k_2 between Zoe and Bob.



$Dec(k_2, C') = M'$

off shore account
say in Panama.
"Panama Papers"

Imagine that Bob represents Bank and Alice is a customer of this Bank. Let Alice has a password to connect to the Bank which is compromised by the Worm infecting its computer It can be done by scanning Alice's keyboard when she is entering a password.

Let Alice wants to transfer a sum of money to her friend Bob2. Then she connects to the Bank and executes KAP described above. But the Alice do not suspect that her computer is infected by the Worm Zoe which realizes MiM attack. So this Worm is in the role of witch. When Alice composes the money transfer document ***M*** to Bob2, she encrypts it by the agreed secret key k_1 using for example AES-128 symmetric encryption scheme by obtaining the following ciphertext

$C = AES\_Enc(k_1, M)$.

Then she sends (she expects that she is sending) ***C*** to the Bank. Ciphertext ***C*** is intercepted by Zoe and sent to its computer. Then Zoe decrypts ***C*** and obtains ***M***

$M = AES\_Dec(k_1, C)$,

and saw the transferring sum and Bob2 account. Then Zoe changes the money transfer account to her account creating a new message ***M`*** and encrypts it with key k_2

$C` = AES\_Enc(k_2, M`)$.

Zoe sends ***C`*** to the Bank.

Bank decrypts ***C`***

$M` = AES\_Dec(k_2, C`)$,

and transfers the indicated sum the Zoe account indicated in ***M`***.

http://crypto.fmf.ktu.lt/xdownload/

- Euronews 17-03-2015 15-38 CET_150316_HTSU_121B0-172837_E.mp4

http://www.euronews.com/2015/03/17/internet-banking-a-hacker-s-ideal-target/

Like Swiss Emmental cheese, the ways your online banking accounts are protected might be full of holes.
According to internet security software developer Kaspersky, the number of cyberthreats reached record levels in 2014. One in three computers or mobile devices were subjected to at least one web attack over the year.
Particular targets are companies or individuals using internet banking.
In January, a Swiss firm lost an estimated one million euros in an online financial transaction that was hacked.
The victim, an accountant at the company, was unaware of what was going on.
It started when he opened an email containing an attachment infected with a virus. Once they had taken control of his computer, all the hackers had to do was wait for him to connect online with his bank.
"When he tried to connect to his bank online, he activated the "Trojan horse". A message appeared asking him to hold. For 20 or 30 minutes, he wasn't able to use his computer at all. During that time, the pirates took control of the computer and carried out several money transfers onto foreign accounts," says Frederic Marchon, spokesman for the Fribourg Police.
Plenty of viruses allowing that kind of illegal activity are available on the internet. The most updated versions are

available for just over 1,000 euros on the darknet.
The hacker gets a warning as soon as someone connects with their bank online using an infected computer.
This IT expert explains how it works: “I can monitor all the computers I have successfully hacked, and I can see precisely, among them, how many are currently banking online and therefore vulnerable. So here, there are two which are currently connected,” says IT expert Cedric Enzler.
Faced with a growing number of cyber attacks on companies, Switzerland has set up an emergency centre to track the attacks and analyse them. But the nature of the centre means they cannot provide with any names or figures.
“It’s a really big problem. You’ve got to realise that anyone who wants to do harm and wants to make money that way will automatically turn to e-banking,” says IT security expert Max Klaus.
For this professor at the Bern University of Applied Sciences, there’s another big problem with this kind of cyber attack: most of the tools we use for internet banking like calculators or smartphone applications designed to read cryptograms are vulnerable to hacking.
“From an electronic point of vue, internet banking is safe. We use secure channels using SSL encryption. The problem comes from the client’s computer, its use no longer guarantees a secure connexion. Whether it’s a computer or a smartphone, hackers can take control and security is compromised,” says Professor Reto Koenig.
None of the banks contacted agreed to answer to our questions on camera.
Swiss banks warn their clients about security problems linked to the use of internet in their general conditions – a warning which often comes with a clause clearing the bank of any responsibility in the event of an attack.
“The client is a victim twice over. First, he’s the victim of a crook, and then he has hardly any chance to defend himself because of the general conditions in his contract. Sometimes, there are agreements between banks and clients but unfortunately, most of the time, these agreements are kept secret, they are confidential, so it’s hard to find out what the procedure is, which is of course detrimental to the client,” says Mathieu Fleury, of the Swiss consumer’s rights association.
A coordinated cyber security taskforce and response scheme, aimed at providing cyber security services for small and medium enterprises in Europe, is to begin pilot deployments in 2015, starting in the UK, the Netherlands and Belgium.
EU authorities are concerned about the vulnerability of SMEs because they employ two-thirds of Europe’s workforce.
More about:

- Banking
- Internet
- Security
- Switzerland

In this report it is pointed out that user, e.g. Alice had a *weak identification* at this time based only on Bank’s passwords submitted to her. While Banks usually have a strong identification based on their public keys certification recognizable by users browsers. The material concerning Public Key Certificates (PKC) we will present later. GPRS
Since that one partial improvement was made by introducing two channel identification based on Smart Id protocol where user must confirm his/her identity using its smart phone and entering pin code.
To provide a strong identification it is required to use cryptographic identification methods together with something like Smart Id and biometrics.
Therefore we start now from cryptographic identification methods and DS schemes.

Smart Id.

Identification: Taiwan GoTrust → μSD

A: PrK = x; PuK = a = g^x mod p: it is infeasible to find x

when p, g, a are given.

Digital signature: to sign a message M and t_A for KAP.

$\sigma_A = sign(x, t_A) = (r_A, s_A)$

$\sigma = sign(x, M) = (r, s)$

A: t_A, σ_A → B: 1) Verifies σ_A on t_A

$\vartheta = Ver(a, \sigma_A, t_A) = \begin{cases} True \text{ „1“} \\ False \text{ „0“} \end{cases}$

2) If $\vartheta =$ „1“ then B executes KAP.

To: For given p, g, a compute x ?

PuK and PrK in general in Public Key Cryptography (PKC) are related with certain funkcija $F(x)$.

$$a = F_{p,g}(x)$$

To compromise PrK=x adersary must be able to compute inverse funkcijs $F^{-1}_{p,g}(a)$:

$$x = F^{-1}_{p,g}(a).$$

Def. Function F is one-way function (OWF) if:
1. computation of direct value $a = F(x)$ is effective.
2. computation of inverse value $x = F^{-1}(a)$ is infeasible.

Ex. Let $a = g^x \bmod p \Rightarrow$ to compute x when p, g, a are given Discrete logarithm Function (DLF) must be applied:

$$\log_g a = \log_g (g^x \bmod p) = x \cdot \log_g g \bmod p = x \cdot 1 \bmod p = x \bmod p \overset{\text{==}}{=} x.$$

when $x < p$

Statement. If $p \sim 2^{2048}$, $|p| \approx 2048$, then Discrete Logarithm Problem (DLP) is infeasible with classical computers.

```
>> p=int64(268435019);
>> g=2;
```

```
>> u=int64(randi(p-1))
u = 79026197
>> tA=mod_exp(g,u,p)
tA = 35 708 470
```

```
>> v=int64(randi(p-1))
v = 26000685
>> tB=mod_exp(g,v,p)
tB = 140166310
```

```
>> kAB=mod_exp(tB,u,p)
kAB = 242789269
```

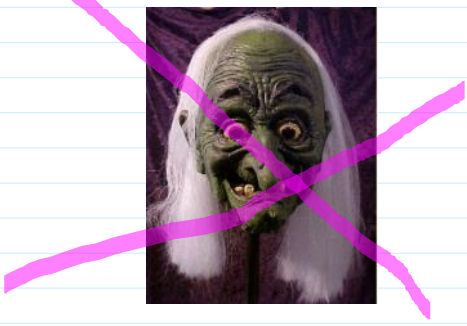
```
>> kBA=mod_exp(tA,v,p)
kBA = 242789269
```

$Sign_{RSA}(d, t_A) = (t_A)^d \bmod n$

RSA signature:

```
>> fy = int64(148374864)
fy = 148374 864
>> d=mulinv(e,fy)
d = 24783857
```

```
>> n = int64(148399231)
n = 148 399 231
>> e = 65537
e = 65537
```



```
>> s=mod_exp(tA,d,n)
s = 110954184
```

$t_A, s \longrightarrow$ B: $(n, e)_A$

$s^e \bmod n = t_A$

```
>> v=mod_exp(s,e,n)
v = 35708470
```